CMS Cliquet Introduction

A constant maturity swap (CMS) cliquet option has two legs: One leg of this deal is based on (regular) floating rates. The other leg links to CMS swap rates. Due to the "set-in-arrear" feature in the structured leg, convexity and timing adjustments have to be considered.

Due to the embedded option and convexity adjustments in the structure leg, we need swap rate volatilities and forward rate volatilities. The former can be interpolated from the implied swaption volatility surface. The latter should be interpolated from the so-called implied forward volatility surface.

However, this surface is currently not available in the market. Therefore, the swap rate volatility surface is also applied to forward rate volatilities. The convexity adjustment does not explicitly depend on the correlation coefficient between the swap rate and the forward rate.

This exogenous parameter is almost impossible to get benchmark value in the market. However, it has been shown that the model provides an upper bound on the break-even floor rate as RBCDS is in the short position in the CMS leg.

It has been recognized that the quantitative analysis of the correlation is not sufficient. Further investigation on estimating the correlation parameters is recommended. For comparison purposes, the implementation of two-factor model of the convexity adjustment could be a good starting point.

Consider a fixed income contract consisting of a regular floating leg and a structured floating leg.

The payoff of the regular leg determined at the *i*th reset date *ti* and paid at the *i*th payment date *ti*+1 is given by

$$P_1 \times (R_i + \mathrm{sp}_i) \times \mathrm{dcf}_i \ .$$

The payoff of the structured leg determined at the *i*th reset date is structured as following:

$$P_2 \times (X_i + \max\{R^{\mathrm{cms}}(s_i) - X_i,\ 0\}) \times \tau_i \ .$$

The present value *V*1 of the regular floating leg of the contract can be easily written as

$$V_1(t_\mathrm{v}) = P_1 \times \sum_{i=1}^{N_1} \left[\mathrm{df}_i^{(1)} \times \mathrm{dcf}_i \times (F_i(t_\mathrm{v}) + \mathrm{sp}_i)\right] \ ,$$

where *Fi*(*t*v) is the forward interest rate seen at the valuation date for the forward period of *Ti* and df*i* is the discounting factor from the *i*th interest payment date *ti*+1 back to the valuation date.

Pricing (see https://finpricing.com/lib/EqWarrant.html) the second leg of the contract is a little bit more involved. Firstly, there is an optionality which is embedded in the contract, and secondly, this leg does not incorporate a natural time lag, which implies that the convexity adjustment is needed.

Let *R*cms(*t; s*) be a forward Δ-term CMS swap rate seen at time *t* and matured at time *s*. Clearly, *R*cms(*si*) = *R*cms (*si; si*). Here we have assumed that the underlying CMS swap structure, which includes the swap term (i.e., ¢), swap frequency *f*, day-count-basis, day-rollover-basis, and holiday-centre, has been specified and given. For each *i* = 1*,…,N*2, let *Bi* be the price at time *si* of a bond with the same structure of the CMS swap and the coupon rate being the initial forward CMS swap rate *R*cms (*t*v*; si*), and *yi* be the yield of the bond, i.e.,

$$B_i = G(y_i) = \sum_{j=1}^{f\cdot\Delta} \left[\frac{\frac{1}{f} R_{\mathrm{f}}^{\mathrm{cms}}(t_{\mathrm{v}}, s_i)}{\left(1 + \frac{y_i}{f}\right)^j} \right] + \frac{1}{\left(1 + \frac{y_i}{f}\right)^{f\cdot\Delta}},$$

where *G*() is the bond price function. We first assume that

$$R^{\mathrm{cms}}(s_i) = y_i \ .$$

Now let *V*2 be the value of this leg and df(2) be the discounting factor from the time *s* back to the valuation date. Then the total payoff of this leg is given by

$$V_2(\bar{s}) = P_2 \times \sum_{i=1}^{N_2} [X_i + \max\{R^{\mathrm{cms}}(s_i) - X_i,\ 0\}]$$

Let E be the expectation operator under the risk-neutral world with respect to the discount bond matured at *s*. Thus the present value of this leg can be given as

$$V_2(t_{\mathrm{v}}) = P_2 \times \mathrm{df}^{(2)} \times \sum_{i=1}^{N_2} \{X_i + \mathrm{E}\left[\max\{R^{\mathrm{cms}}(s_i) - X_i,\ 0\}\right]\}$$

We further assume that

$$\ln R^{\mathrm{cms}}(s_i) \ \sim\ \mathrm{N}\left[\ln \mathrm{E}[R^{\mathrm{cms}}(s_i)] - \frac{\sigma_i^{\mathrm{cms}2}\delta_i^{\mathrm{s}}}{2},\ \ \sigma_i^{\mathrm{cms}2}\delta_i^{\mathrm{s}}\right]$$

If we can calculate the expectation of the future CMS swap rate, then present value (7) can be obtained by using Black's formula. Considering the convexity adjustment, we can approximate the expectation as follows:

$$\mathrm{E}[R^{\mathrm{cms}}(s_i)] \approx \left[y_i^{(0)} - \frac{1}{2} {y_i^{(0)}}^2 {\sigma_i^{\mathrm{cms}}}^2 \delta_i^{\mathrm{s}} \frac{G''(y_i^{(0)})}{G'(y_i^{(0)})} \right] \times \exp\left(- \frac{F_i^{(0)} \rho_i \sigma_i^{\mathrm{cms}} \sigma_i^{\mathrm{fwd}} \delta_i^{\mathrm{s}} \delta_i^{\mathrm{f}}}{1 + F_i^{(0)} \delta_i^{\mathrm{f}}} \right) = m_i \ ,$$

We can see that another piece of information which we can not get benchmark values in the market is the correlation parameters. Roughly, we may conclude that the CMS swap rates are positively correlated with those forward rates.

Furthermore, we can see that overlap periods of underlying CMS swaps and forward rate are decreasing as *i* increases from 1 to *N*2. It may imply that the correlation coefficient *½i* should be decreasing with respect to *i*. However, the detailed information of these correlation coefficients is unknown to us.

An alternative is the single factor model. In this approach, a fixed correlation coefficient is embedded in the convexity adjustment. In the case when we set the correlation coefficient to unity and apply linear approximation to the function of *ex*, the results are identical from the two approaches.

Based on the current market information and the understanding of the dynamics of CMS swap rates and forward interest rates, there is no clear answer which approach is correct. The best we can do at this moment is to test several cases of variant correlation coefficient settings.